PHENOMENOLOGY OF COMPACTIFICATIONS OF M-THEORY ON MANIFOLDS OF G2 HOLONOMY

by

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I. Introduction

The Standard Model is a phenomenally successful theory of particle physics, well tested at quantum mechanical level up to the TeV energy scale, and renormalizable (notwithstanding that the Higgs particle has not been observed yet) [1].

However there are several issues (or at least perceived ones) with the model. First of all, it only describes three of the four fundamental forces (the strong, electromagnetic and weak forces) in a quantum manner, leaving gravity still described only in a classical fashion; however we will not be concerned with this any further during this dissertation.

The primary issue with which we will be concerned is the so-called hierarchy problem (also known as the weak-scale instability) – why there are two (electroweak and Planck) mass scales and why they are so far apart, why the electroweak scale is not modified at loop level by the Planck scale and whether there are any other scales between them. [1,2]

One wonders why we should be concerned with as yet purely theoretical imperfections which have not yet manifested themselves in any experimental observations. One only has to look to the history of the standard model itself for a precedent for looking for physics beyond it – the earliest theories of the weak interaction, which were based on the interaction of four fermions at a point, broke down when calculated to higher order at the then unimaginably high energies of 300 GeV (Heisenberg 1939) before they had been violated by any observation. This eventually resulted in the development of the Weinberg-Salam theory of the electromagnetic and weak interactions with spontaneous symmetry breaking in the form of the Higgs mechanism giving masses to W and Z bosons, leptons and quarks (indeed all the masses in the standard model). Therefore it seems reasonable to be concerned with theoretical imperfections in and weaknesses of the standard model, even before they are found wanting experimentally. [1]

The standard model is renormalizable, which means finite results are obtained for loop corrections, even when internal momenta are taken to infinity. However it is generally believed that there is physics beyond the standard model (i.e. at higher energies), which means the cutoff scale cannot be taken to infinity (if this were not the case and the standard model was indeed the whole story, then the ‘bare’ Higgs mass and coupling terms could be redefined in terms of the cutoff as in the normal procedure of renormalization, and so there would be no issue). [1]

Supersymmetry, in which each particle has a corresponding ‘superpartner’ of opposite type (boson/fermion), is one way proposed to solve the hierarchy problem. It is based on the fact that closed fermion loops contribute a factor of -1 to a Feynman diagram, and therefore the loop divergences that result from a boson loop are cancelled out when the loop bosons are replaced by their fermion superpartners and vice versa. This can explain, for example, why the Higgs particle, the only scalar in the standard model, does not have corrections to its ‘bare’ mass of order the Planck scale. (This is only an issue with scalar particles; the masses of chiral fermions and gauge bosons are constrained to be zero by chiral and gauge symmetry respectively, though they (may) acquire masses by mechanisms akin to Higgs’, and the same symmetries constrain the loop corrections to their mass to be proportional to it and have only logarithmic divergences in the cutoff. [1,2]) The simplest extension to the standard model containing all of its particles and all of their superpartners is called the minimal supersymmetric standard model (MSSM). (We note it must contain two Higgs doublets, one coupling to the negatively charged quarks and one to the positively charged ones, since if there were just one, one type of quark would have to couple to the complex conjugate of the Higgs field coupling to the other type and this is not allowed by supersymmetry. [1])

If supersymmetry exists, it must be with N=1, otherwise there can be no chiral fermions, because then the states of helicity -1/2 (left handed) and +1/2 (right handed) must be in the same supermultiplet and hence transform in the same way under the gauge group, which is not the case, as the left-handed fermions transform as doublets of the SU(2) part (isospin for quarks and weak isospin for leptons), but the right-handed ones transform as singlets. (Since massive particles must travel at strictly less than the speed of light, they therefore must exist with both helicities, meaning that the left and right handed forms must interact to give the particles their masses, this is the reason why explicit mass terms for fermions cannot be incorporated into the Lagrangian and their masses must result from spontaneous symmetry breaking, hence the Higgs model mentioned above.)

This supersymmetry is not observed in nature, so it must be broken at some energy scale, which may be as low as the TeV scale but obviously cannot be lower because superpartners have not been observed yet. There are two ways to break this symmetry, firstly via spontaneous symmetry breaking akin to the Higgs model, and secondly by adding explicit supersymmetry breaking terms, so-called ‘soft’ terms. Currently the second method is the more successful; of course it does not give exact cancellation of the Feynman diagrams with loops of a particular boson and those of the fermion superpartner (and vice versa), but it does not re-introduce the quadratic divergences that supersymmetry removed, instead giving only logarithmic ones. It even gives the result that for the electroweak mass scale to be the observed value of order 100 GeV, the supersymmetry breaking scale should be of order 1 TeV as we will seek to observe at the Large Hadron Collider now. [1,2]

Supersymmetry, however, leaves many questions unanswered, hence one considers promoting it to a local gauge symmetry, called supergravity, by introducing a spin-2 field called the graviton, along with its spin-3/2 superpartner, the gravitino, and coupling them appropriately to the chiral and vector supermultiplets already in the theory. It is phenomenologically attractive, since explicit ‘soft’ supersymmetry breaking terms manifest themselves at low energy scales (below the Planck scale) as a result of spontaneous supersymmetry breaking at higher energy scales. One usually introduces one or more ‘hidden’ sectors at higher energies (typically around 10^{12} GeV) which are coupled to the ‘visible’ sector of the MSSM only via gravity; these transfer supersymmetry breaking to the MSSM at a scale of roughly 1 TeV as desired. [2]

However, even supergravity is incomplete, still being purely classical in its description of gravity and being incomplete at the ultraviolet level, since it is neither finite nor renormalizable. [2] The only known ultraviolet completion of supergravity is superstring theory. [13] There are several different superstring theories, namely type I, type IIA, type IIB and the two heterotic string theories (based on the SO(32) and E8xE8 groups respectively), all in 10 dimensions. M-theory originally arose as a limit of type IIA string theory at strong coupling, as the string ‘grew’ an 11^{th} dimension via the dilaton (and conversely type IIA string theory is the same as M-theory compactified on a circle); the same thing was later observed to happen with the E8xE8 heterotic string (akin to compactification on a line, or a circle orbifolded out by Z2), and still later, via dualities, the other superstring theories were similarly also established as limits of M-theory under different conditions, as was 11-dimensional supergravity. The full formulation of M-theory has not yet been established, but this in no way precludes its great usefulness as a theory, though it is not technically a string theory since its low-energy limit does not contain strings. [3]

Superstring theories have 10 dimensions and M-theory 11, but we observe a 4-dimensional universe – something must be done to bridge the gap between the two. The most historically studied approach is Kaluza-Klein compactification, in which one or (as here) several dimensions are reduced to finite size and identified periodically or in similar fashion. This approach is sometimes extended to dimensional reduction, in which only the massless modes in the compact dimensions are kept, which is the case when the energy scale is below the scale specified by their size, noting that the solution of the dimensionally reduced theory must also be a solution of the full theory. (This is consistent when the manifold on which the reduction is performed is a circle or d-torus, since they are singlet representations of the U(1) (isomorphic to SO(2)) symmetry group relating the modes to each other, while the massive modes form doublet representations (m,-m), and products of singlets either with singlets or non-singlets cannot produce new non-singlets absent from the original representations. It is not always the case for more complicated manifolds, but it turns out that the cases where it is so include the cases of greatest interest in string theory and M-theory. [7])

In this dissertation we shall first introduce the concept of holonomy, a property of manifolds and connections that determines the supersymmetric properties of the manifold. We will then discuss the Kaluza-Klein compactification of M-theory on a 7-dimensional compact manifold with holonomy group G2, which we will see to be necessary for the theory to be phenomenologically viable since it is the only case starting from M-theory that gives a 4-dimensional theory with N=1. We will next cover the need for the manifold to be singular to give the observed spectrum of standard model particles (never mind as-yet-unobserved supersymmetric or other particles), and lastly deal with the problem of the massless scalars that result from the compactification.

II. Holonomy and G2 manifolds

The holonomy group of a manifold (or more correctly of a connection on a manifold, since different connections, e.g. the Levi-Civita connection and the spin connection, can have different holonomies) is the group under which various quantities (vectors, tensors, spinors etc.) transform under parallel transport around a closed path on the manifold. For a d-dimensional Riemannian manifold, it is always a subgroup of SO(d) if the quantity is a tensor (including the case of a vector), and of Spin(d) (the double cover of SO(d)) if it is a spinor. (Technically the holonomy group must first be defined separately for each point on the manifold, however if the manifold is connected then the holonomy group is trivially independent of the base point and can therefore be defined for the whole manifold. [6])

Actions are generally written with only those terms in the bosonic fields included – the reason for this is that they are used to construct classical solutions of the theory, in which the fermionic fields vanish. Supersymmetric variations of the bosonic fields include at least one fermionic field in each term, and hence vanish identically classically; therefore it is necessary only to consider the supersymmetric variations of the fermionic fields, and then only the terms which involve only bosonic fields. [3] However the full 11-dimensional supergravity action with fermionic terms included is well-defined and given in [9].

'Ordinary' toroidal compactification preserves all the supersymmetry (i.e. keeps the same number of supercharges) of the original theory, since the holonomy group is trivial – a quantity is not changed by parallel transport even along a non-closed path. This means that since the number of supercharges (16 – in the form of one 10-dimensional Majorana-Weyl spinor – in type I and the two heterotic theories, 32 – 2 such spinors – in the two type II theories and again 32 – this time one 11-dimensional Majorana spinor – in M-theory) remains the same, the number of supersymmetries will increase on compactification, since the dimension of the minimal spinor decreases. In particular, the minimal spinor in 4 dimensions (using Lorentzian signature, i.e. SO(3,1)) has 4 real components – the 4 complex components of the Dirac spinor become 8 real ones, and this is halved by imposing either the Majorana or Weyl condition (for d=0 mod 4 the Weyl representations of SO(d-1,1) are complex conjugate to each other, not self-conjugate, so they cannot also be made Majorana).

[4] Therefore, when compactified down to 4 dimensional Lorentzian-signature space, there are 4 supersymmetries (so N=4) in the type I and heterotic theories and N=8 in the type II theories and M-theory.

By contrast a generic connection on the compact manifold will have the full SO(d) or Spin(d) as its holonomy group, and since the spinor representation of this group is irreducible, it contains no singlets under the group action, hence there is no spinor that is preserved under parallel transport (along a generic closed loop), i.e. no covariantly constant spinor, and so no supersymmetry is preserved, N=0.

An important question is whether we can find a 'halfway house' case, where some but not all of the supersymmetry is preserved under the compactification. This requires the holonomy group to be a proper subgroup of SO(d). One way to do this is to use orbifolds, where points on the manifold are identified under a discrete symmetry such as a cyclic group; this however gives a discrete holonomy group isomorphic to the cyclic group when the path encloses a fixed point of the orbifold transformation group, and the trivial holonomy group when it does not (it is possible for different points to have different holonomy groups since the orbifold is not simply connected). Another way, which is the one we will explore, is to use so-called manifolds of special holonomy, of which G2 manifolds are a subset and the only ones which are 7-dimensional as required here, indeed the only odd-dimensional ones. (Other examples, which are all even-dimensional, are Kahler, Calabi-Yau, hyper-Kahler and Spin(7) manifolds that we will mention later.)

The number of supersymmetries (N) preserved for a specified background is given by the number of Killing spinors under the connection on the manifold (an analogue of Killing vectors, which parametrize infinitesimal bosonic symmetries). When all bosonic fields apart from the metric are set to zero, i.e. there are no fluxes, a Killing spinor is the same as a covariantly constant spinor, and the number of supersymmetries is thus the number of singlets that arise when the spinor representation of SO(d) or Spin(d) is decomposed into representations of the holonomy group of the connection on the manifold.

When 11-dimensional space is decomposed into a product of a non-compact 4-manifold and the (assumed to be compact here) 7-dimensional G2-manifold, an 11-

dimensional 32-component spinor can be decomposed into the product of a 4-component 4-dimensional spinor function of the noncompact dimensions and a 7-dimensional 8-component spinor function of the compact ones and the condition of 'Killingness' and/or covariant constancy can be split up into independent conditions on the two spinors.

[Under SO(10,1) -> SO(3,1) x SO(7): 32 = (4,8).

Considering now only the 8 of SO(7):

For SO(7), the Cartan Matrix is given by:

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[ 2 -1  0]
[-1  2 -2]
[ 0 -1  2]
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The spinor ([0 0 1]) representation in Dynkin Basis is given by:

[0 0 1], [0 1 -1], [1 -1 1], [-1 0 1], [1 0 -1], [-1 1 -1], [0 -1 1], [0 0 -1]

For G2, the Cartan Matrix is given by:

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[ 2 -3]
[-1  2]
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The fundamental ([0 1]) representation in the Dynkin Basis is given by:

[0 1], [1 -1], [-1 2], [0 0], [1 -2], [-1 1], [0 -1]

Transforming the representation of SO(7) into reps of G2 using the rule [x y z] -> [y x+z]:

[0 1], [1 -1], [-1 2], [0 0], [0 0], [1 -2], [-1 1], [0 -1]

This splits up into the fundamental ([0 1]) representation and the trivial ([0 0]) one:

8 -> 7 + 1]

The existence of a covariantly constant spinor only implies that the holonomy group is contained in G2, not that it is exactly G2. For there to be N=1 supersymmetry, i.e. only one covariantly constant spinor, the holonomy group must be exactly G2, since there is no subgroup of G2 under which the fundamental 7 representation does not decompose into a combination containing at least one more singlet (e.g. decomposing under SU(3), one gets a 3, a $\bar{3}$ and a 1).

When there are no fluxes, the existence of a covariantly constant spinor implies that the manifold is Ricci-flat, since the commutator of two covariant derivatives acting on the spinor, which is trivially zero, gives the Riemann tensor times a 2-index gamma matrix times the spinor; contracting this with a 1-index gamma matrix gives a term which vanishes by the 'bastard' symmetry and another term which is the Ricci tensor times a gamma matrix times the spinor; since the left-hand-side (the commutator) vanishes, this product must also vanish for all possible indices of the Ricci tensor and thus the Ricci tensor itself must vanish.

By Berger's classification theorem, G2 is the only possible reduced (i.e. not trivial or the full SO(d)) holonomy group for an odd-dimensional manifold. (Indeed the only possible reduced holonomy groups even on even-dimensional manifolds, if the manifold is simply connected and locally neither a product nor a symmetric space – which is necessary since symmetric spaces admit an inversion symmetry and cannot therefore be supersymmetric [5] – are very restricted, comprising only Kahler manifolds with holonomy group U(d/2), Calabi-Yau (SU(d/2)), hyper-Kahler (Sp(d/4)), G2 in dimension 7 and Spin(7) in dimension 8. Only the Calabi-Yau, G2 and Spin(7) cases are Ricci-flat. [6])

The compactification of M-theory on G2 manifolds has not been studied very much until recently, because it is not easy to explicitly construct of metrics with G2 holonomy, nor is there even an existence theorem akin to the one for Calabi-Yau manifolds [8], nor a corresponding ready application of the techniques of algebraic geometry to their study. [6] However examples certainly do exist, the first compact ones were constructed by Joyce [11], which he did by orbifolding the 7-torus by a finite group which preserves a 3-form called a calibration just as the full G2 group does (in fact both the G2 group and G2 manifolds are usually defined in terms of the calibration rather than the metric, though it is not so easy to relate this to the decomposition of the spinor representation), and then resolving the singularities by cutting out a ball around each one and replacing it with a so-called Eguchi-Hanson space, giving a smooth G2 manifold. [3] Kovalev [12] subsequently constructed new examples by gluing together two non-compact asymptotically cylindrical Riemannian manifolds with holonomy SU(3). [6] Grigorian [6] showed that given a calibration, one could construct a metric. (Given a (Killing or) covariantly constant spinor, it is trivial to construct such a calibration. [5])

All these examples are smooth; singular examples, which are necessary to give non-abelian gauge groups and chiral fermions (see the next section), can be implied by duality with the heterotic and type IIA theories compactified on Calabi-Yau 3-folds [14].

These examples refer to backgrounds without flux; there are ways of turning fluxes on to 'convert' non-compact G2 manifolds into compact manifolds with so-called 'weak holonomy' group G2, but this leads to unphysical anti-de Sitter spacetime in the four non-compact dimensions, though this can be disregarded when dealing with such topics as anomaly cancellation, since these are topological properties. (In any case 'weak holonomy' is not good terminology, since the holonomy does not apply so much to the manifold itself as to the connection on the manifold, and in supersymmetric theories it generally refers to the supersymmetric connection containing both Levi-Civita and spin components, as opposed to the traditional definition using just the Levi-Civita connection. [5]) It is also the case that while no compact G2 manifolds have been constructed yet with conical singularities, the principle of 'weak holonomy' has been used to transform non-compact G2 manifolds which are asymptotically conical into compact 'weak G2 manifolds' with two conical singularities. [16]

In the next section we will discuss the types of matter that result from compactification of M-theory on G2 manifolds, and show the requirement that these manifolds must be singular for phenomenologically realistic particle spectra to occur.

III. Chirality and singular manifolds

M-theory, being odd-dimensional, does not have chiral fermions (which are specified by chiral spinor representations of a group), since the Γ11 (or Γ10 depending on convention) matrix, which commutes with all the generators of the spinor representation of SO(9,1) and thus splits the 32-(complex) component Dirac representation into two Weyl representations each with 16 components and opposite eigenvalues under Γ11 which do not change under SO(9,1) transformations (using Schur's lemmas), anticommutes with the remaining generators of SO(10,1) and therefore there is only one spinor representation of the group.

This means that it cannot have chiral fermions either when compactified on a smooth manifold, even though the remaining SO(3,1) group does admit them. It is therefore necessary to have singularities or other defects on the manifold if chiral fermions are to result. [3]

Another problem, and therefore another reason for requiring singularities on the manifold, is that when the manifold is not only smooth but large in comparison to the Planck scale, which is the only case for which 11-dimensional supergravity adequately describes the situation, the gauge fields are all abelian and also the massless matter multiplets are all uncharged. [8] This results from the fact that the manifold has no continuous symmetries, because it has no Killing vectors to generate them; the Ricci-flatness of the manifold means that Killing's equation implies Laplace's equation, and hence that the Killing vector is covariantly constant, but the vector representation of SO(7) is irreducible when decomposed into representations of G2, so this cannot happen. [9] The only gauge fields are therefore those resulting from dimensional reduction of the 3-form in the action; they are abelian and do not couple to any charged matter fields at all in the supergravity approximation. [8]

[The SO(7) vector ([1 0 0]) representation in the Dynkin basis is given by:

[1 0 0], [-1 1 0], [0 -1 2], [0 0 0], [0 1 -2], [1 -1 0], [-1 0 0]

Doing the same transformation into G2 representations as for the spinor representation, one gets:

[0 1], [1 -1], [-1 2], [0 0], [1 -2], [-1 1], [0 -1]

This is the fundamental G2 representation so the SO(7) vector representation is irreducible under G2.]

However, while the low-energy supergravity limit is only valid for smooth manifolds which are large in relation to the Planck scale, the full M-theory, though it has not been completely formulated yet, does admit the possibility of compactifications on G2 manifolds with singularities, which could yield the required chiral fermions in Minkowski spacetime and realistic non-abelian gauge groups. [5]

Dualities exist between M-theory compactified on a G2 manifold and the heterotic string (E8xE8) compactified on a Calabi-Yau 3-fold, which is 6-dimensional; however since the heterotic string theory is chiral, having only one (Majorana-)Weyl spinor (in contrast to M-theory in which, being odd-dimensional, there is no Weyl decomposition into chiral spinors), chiral fermions and non-abelian gauge fields can result from this latter compactification even when the Calabi-Yau 3-fold is smooth. There is, however, one definite example where the heterotic E8xE8 string compactified on a Calabi-Yau 3-fold gives a non-chiral theory, and that is when the Euler number of the 3-fold vanishes; this occurs when the second and third Betti numbers of the 3-fold are 16 and 39 respectively. Remarkably, considering the duality between the two compactifications relates manifolds with the same such numbers, G2 manifolds with these Betti numbers do exist.

The best explanation for non-abelian gauge symmetries comes from further dualities between the heterotic E8xE8 string compactified on a 3-torus and M-theory compactified on K3, a 4-dimensional Calabi-Yau 2-fold (one of only two such 2-folds, and indeed the only one which has exactly SU(2) as its holonomy group). The singularities of this 4-dimensional manifold occur at the fixed points of a finite subgroup of SU(2); the specific group is specified by the intersections of supersymmetric 2-cycles on the manifold, any two such cycles can intersect once or not at all which gives a pattern similar to a simply-laced (where all the lines are single, i.e. all the roots are the same length) Dynkin diagram. Remarkably, since this similarity is wholly coincidental, the two concepts being entirely different, the Dynkin diagram of the resulting non-abelian gauge group is exactly the same as the diagram showing how the supersymmetric 2-cycles on the manifold intersect. (Since these non-abelian gauge groups are called An (n>0), Dn (n>3) and En (n=6,7,8), the corresponding finite subgroups of SU(2) giving these groups are given the same names, hence the ADE classification.) [3]

Chiral fermions are explained by more exotic singularities where the metric of the manifold is locally described by an (isolated) conical singularity, which can occur when a supersymmetric cycle (usually a 3-cycle) shrinks to zero size. The details of the singularities can be worked out using the aforementioned dualities with the heterotic E8xE8 string compactified on a Calabi-Yau 3-fold. [18]

IV. Moduli stabilization and the hierarchy problem

A major problem with compactification and dimensional reduction is that the effective action in the reduced number of (non-compact) dimensions contains massless scalars which represent quantities such as the dimensions of the compact space, the metric on this space and (if any fluxes are present) their components in the compact directions. The scalars related to the compact metric come from the splitting of the Ricci scalar into the two sets of directions; both they and the scalars related to fluxes arise because all bosonic terms in the original 11-dimensional supergravity action contain exactly two spacetime derivatives as they must do by dimensional analysis and so therefore must the dimensionally reduced action. [3]

These scalars are similar to Goldstone bosons in that they are massless, since there is no potential in the action; this makes states with different values of the scalars degenerate. However, they are different in that there is no symmetry relating degenerate states with each other and thus the physics depends on the values of the scalars. In bosonic string theory the degeneracy is accidental and broken by the one-loop energy, but in supersymmetric string theories the existence of such degenerate but inequivalent vacua is common and is important in understanding the dynamics of the system. These scalars, which are called moduli, have not been observed in nature; furthermore if they existed they would mediate gravity-like long-range interactions (notwithstanding that the graviton has not been observed either yet). All moduli must therefore acquire masses through the terms in the Lagrangian that break supersymmetry [10]; this is called moduli stabilization, whereby the moduli masses are the vacuum expectation values (VEVs) of moduli fields akin to the as yet undiscovered Higgs field. We shall cover this topic in this section.

It is also true that, since string theory has no dimensionless parameters, the values of the standard model masses and coupling constants can only be described by moduli. [14, 15] This means that the hierarchy problem is a double-edged sword; one has to both stabilize all the moduli and generate the hierarchy simultaneously.

There has been much progress in recent years in understanding the mechanisms which do this. One method of stabilizing moduli involves the use of fluxes, i.e. having non-zero values for the field strengths appearing in the Lagrangian (in M-theory this is a 4-form F, which is here the exterior derivative of a 3-form gauge field A; in other theories it may have different dimensions). This method has been well understood in the case of type IIB string theory, where the fluxes have odd dimension and their number is large in comparison with the number of moduli so the hierarchy can be generated by fine-tuning them; by contrast this is not possible in the type IIA and heterotic string theories and M-theory because the number of fluxes is comparable to the number of moduli. This means that the superpotential is large, and hence so is the gravitino mass (acquired by swallowing a modulus as a 'Goldstino') unless the extra dimensions have a large volume, and furthermore all scalar masses; in this case the weak scale is either zero or a large value comparable to the Planck scale (it is smaller in the heterotic theory, but not by nearly enough, by only a few orders of magnitude), so compactification using fluxes does not solve the hierarchy problem. One therefore must attack the problem from an alternative angle, compactifying without fluxes. [14,15,17]

Each modulus has an axionic superpartner, which means that the supermultiplets have a so-called Peccei-Quinn shift symmetry, which does not occur in theories other than M-theory (i.e. the five actual superstring theories). This can only be broken by non-perturbative effects, and so the superpotential is entirely non-perturbative. This can arise from strong gauge dynamics in two non-abelian, asymptotically free gauge groups, one of which contains the visible sector and the other the so-called hidden sector(s). In general the superpotential can depend upon all the moduli, in which case one might expect that they are all stabilized; this is indeed the case.

(The hidden sector(s) as mentioned in the introduction are sectors of a given model that couple to the standard model particles only through gravitational interactions. In the E8xE8 heterotic string theory it is generally understood to mean the second E8 (with the first one containing the standard model particles). In M-theory it is not usually so easy to define how they are decoupled from the visible sector, but the term is still used; moreover since chiral fermions arise from locally conical singularities on the G2 manifold, these singularities arise when a supersymmetric 3-cycle shrinks to zero size and (because 2x3<7) two such cycles generally do not intersect, the sector not containing the standard model can define a hidden sector(s) and supersymmetry breaking will be mediated solely by gravitational interactions. (Since the E8xE8 heterotic theory is compacted on a 6-dimensional space, two such cycles in this space will generally intersect at at least a point, and so supersymmetry breaking will be gauge-mediated, though of course the hidden sector(s) are in general defined differently as (being part of) the second E8.))

One may start by considering just a single hidden sector; while this does in fact stabilize all the moduli, it is non-generic and fixes them in a region far beyond the validity of the supergravity approximation [17], so from now on we will consider only those cases where there are at least two hidden sectors, one of which we take to contain one flavour of quarks. [15]

The G2-MSSM gives a distinctive spectrum, where the gauginos are 'light' and suppressed relative to the scalars (including sfermions) and Higgsinos, which have a mass of between 10 and 100 TeV (the lower limit being set by the as yet unobserved nature of the gauginos, and by the need to solve the moduli and gravitino problems which relate to the early universe in the early post-inflation era) by a factor of roughly 83, which comes from a formula in the dimensions of the gauge group of the quark-containing hidden sector which we take to be SU(Q)xSU(P+1) for some positive integers Q and P assuming one flavour of quarks; this constrains the gauge group because it must be at least equal to the number of moduli, which must be large to accommodate the over 100 couplings that occur in the MSSM. [15]

Other distinctive features of the G2-MSSM are the markedly lower mass of the stop squark compared with the other squarks, which is caused by renormalization group effects, and the primarily wino (the superpartner of a W-like particle, i.e. a SU(2) gaugino) nature of the lightest supersymmetric particle (LSP), which is thought to be a primary component of dark matter (while the type IIB theory gives an exclusively bino, i.e. a U(1) hypercharge gaugino, LSP). [15] The LSP would be stable, owing to conservation of a multiplicative quantity called R-parity, where supersymmetric particles have R-parity -1 and 'normal' (including the standard model) particles have R-parity +1. One generic problem in string and M-theory compactifications is the issue of the decay of moduli happening at later times than expected owing to the weakness of their (purely gravitational) coupling to the visible sector; this spoils the so-far successful predictions of Big Bang Nucleosynthesis; in the G2-MSSM this is solved because since all the moduli are stabilized, the moduli mass matrix and their couplings to the visible sector can be explicitly calculated in terms of certain 'microscopic' parameters, and this gives one modulus much heavier than all the others, the latter having masses of order that of the gravitino. [15]

One remaining issue with the G2-MSSM is the fact that the mass of the Z boson must be fine-tuned, this is called the little hierarchy problem and it is a more serious problem than the 'primary' hierarchy problem because of the larger scalar masses. [15] Another is the size and sign of the cosmological constant. Further work will be required to resolve these issues.

V. Conclusion

While the standard model is a phenomenally successful theory of particle physics, it has clearly apparent shortcomings, notably the absence of both a consistent quantum theory of gravity and an explanation for the scale of the masses of its particles; progressively resolving these shortcomings leads eventually to superstring theory in its various guises, and hence to M theory as the leading candidate(s) for a so-called 'Theory of Everything'. However, these latter theories are 10- and 11-dimensional respectively, while we observe a 4-dimensional universe; hence, now considering only the M-theory case, we must compactify seven of these dimensions down to finite size to get the four dimensions we see. The requirement of chiral fermionic matter firstly constrains the compact 7-dimensional manifold to have the exceptional group G2 as its holonomy group, and subsequently forces the manifold to be singular in a particular way. This compactification however produces massless scalars called moduli which are not observed in nature, so they must be made massive in some way which generates and stabilizes the mass scale of the standard model relative to the Planck scale at which quantum gravitational effects are expected to arise. This is called the hierarchy problem.

Since compactifications on manifolds with holonomy group G2 using fluxes fail to solve the hierarchy problem, we must instead compactify on such manifolds without using fluxes; this method does indeed generate and stabilize the hierarchy, which makes it a valid theory against which to compare the observed signatures from the Large Hadron Collider; the expected spectrum if the G2-MSSM is indeed the correct theory is distinctive, with light gauginos, heavy gravitinos, a much lighter stop squark than the other squarks and a wino lightest supersymmetric particle.

There are still outstanding issues and therefore opportunities for future research, in particular the little hierarchy problem involving the Z-boson mass, the size and sign of the consmological constant and the difficulty in constructing manifolds of G2 holonomy with the right singularity properties; however since the G2-MSSM does solve the primary hierarchy problem, its predicted signatures can legitimately be compared with the results from the LHC and therefore the theory already provides testable predictions. With the mushrooming volume of data being produced by the LHC, these predictions can be tested sooner rather than later.

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